CBCS Scheme

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15MAT31

Third Semester B.E. Degree Examination, June/July 2017
Engineering Mathematics - III

Time: 3 hrs. Max. Marks: 80

Note: ***Answer FIVE full questions, choosing one full question from each module.***

Module-1

1 a. Obtain the Fourier series expansion of

$$f(x) = \begin{cases} \pi x & 0 \le x < 1 \\ \pi(2-x) & 1 \le x \le 2 \end{cases}$$ **(08 Marks)**

and deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots = \frac{\pi^2}{8}$.

b. Obtain the constant term and first sine and cosine terms in the Fourier expansion of y from the following table. **(08 Marks)**

x	0	1	2	3	4	5
y	9	18	24	28	26	20

OR

2 a. Expand $f(x) = |x|$ as a Fourier series in $-\pi \le x \le \pi$ and deduce that **(06 Marks)**

$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots = \frac{\pi^2}{8}$.

b. Obtain the half range cosine series for the function $f(x) = x \sin x$ in $0 < x < \pi$. **(05 Marks)**

c. The following table gives variations of periodic current over a period T. Show that there is a direct current part of 0.75 amp in the variable current and obtain the amplitude of first harmonic. **(05 Marks)**

t(sec)	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$
A (amp)	1.98	1.3	1.05	1.3	-0.88	-0.25

Module-2

3 a. Find the Fourier Transform of

$$f(x) = \begin{cases} 1 - x^2 & |x| \le 1 \\ 0 & |x| > 1 \end{cases}$$ **(06 Marks)**

Hence evaluate $\int_0^\infty \frac{x\cos x - \sin x}{x^3} \cos \frac{x}{2}\, dx$.

b. Find the Fourier cosine transform of

$$f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2 - x & \text{for } 1 < x < 2 \\ 0 & \text{for } x > 2 \end{cases}$$ **(05 Marks)**

c. Find the inverse Z – transform of

$$\frac{3z^2 + 2z}{(5z-1)(5z+2)}.$$ **(05 Marks)**

OR

4 a. Find the Fourier sine transform of $\frac{e^{-ax}}{x}$, $a > 0$. **(06 Marks)**

b. Find the Z – transform of i) $\cosh n\theta$ ii) n^2. **(05 Marks)**

c. Solve the difference equation $y_{n+2} + 4y_{n+1} + 3y_n = 3^n$ with $y_0 = 0$, $y_1 = 1$. **(05 Marks)**

Module-3

5 a. Find the coefficient of correlation and two regression lines for the following data : **(06 Marks)**

x	1	2	3	4	5	6	7	8	9	10
y	10	12	16	28	25	36	41	49	40	50

b. Fit a curve of the form $y = ae^{bx}$ for the following data : **(05 Marks)**

x	5	6	7	8	9	10
y	133	55	23	7	2	2

c. Use Newton – Raphson method to find a real root of the equation $x \sin x + \cos x = 0$ near $x = \pi$. **(05 Marks)**

OR

6 a. In a partially destroyed lab record, only the lines of regression of y on x and x on y are available as $4x - 5y + 33 = 0$ and $20x - 9y = 107$ respectively. Calculate $\bar{x}$, $\bar{y}$ and coefficient of correlation between x and y. **(06 Marks)**

b. Fit a second degree parabola to the following data : **(05 Marks)**

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y	1.1	1.3	1.6	2.0	2.7	3.4	4.1

c. Use the regula – falsi method to obtain a root of the equation $2x - \log_{10}x = 7$ which lies between 3.5 and 4. Carryout 2 iterations. **(05 Marks)**

Module-4

7 a. The population of a town is given by the table **(06 Marks)**

Year	1951	1961	1971	1981	1991
Population in thousands	19.96	39.65	58.81	77.21	94.61

Using Newton's forward and backward interpolation formula, calculate the increase in the population from the year 1955 to 1985.

b. Use Lagrange's interpolation formula to find y at x = 10, given **(05 Marks)**

x	5	6	9	11
y	12	13	14	16

c. Given the values

x	2	4	5	6	8	10
y	10	96	196	350	868	1746

Construct the interpolating polynomial using Newton's divided difference interpolation formula. **(05 Marks)**

OR

8 a. From the following table, estimate the number of students who obtained marks between 40 and 45. **(06 Marks)**

Marks	30-40	40-50	50-60	60-70	70-80
No. of students	31	42	51	35	31

b. Apply Lagrange's formula inversely to obtain the root of the equation $f(x) = 0$, given $f(30) = -30$, $f(34) = -13$, $f(38) = 3$, $f(42) = 18$. **(05 Marks)**

c. Use Simpson's $\frac{1}{3}$ rule to find $\int_0^{0.6} e^{-x^2} dy$ by taking 7 ordinates. **(05 Marks)**

Module-5

9 a. Find the work done in moving a particle in the force field $\vec{F} = 3x^2\, i + (2xz - y)j + z\, k$ along the curve defined by $x^2 = 4y$, $3x^3 = 8z$ from $x = 0$ to $x = 2$. **(06 Marks)**

b. Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)i - 2xy\, j$ around the rectangle $x = \pm a$, $y = 0$, $y = b$. **(05 Marks)**

c. Solve the Euler's equation for the functional $\int_{x_0}^{x_1} (1 + x^2 y^1) y^1 dx$. **(05 Marks)**

OR

10 a. Verify Green's theorem for $\int_c (xy + y^2)dx + x^2 dy$, where c is bounded by $y = x$ and $y = x^2$. **(06 Marks)**

b. Evaluate the surface integral $\iint_s \vec{F} \cdot N ds$ where $\vec{F} = 4xi - 2y^2 j + z^2 k$ and s is the surface bounding the region $x^2 + y^2 = 4$, $z = 0$ and $z = 3$. **(05 Marks)**

c. Show that the shortest distance between any two points in a plane is a straight line. **(05 Marks)**

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